QUANTITATIVE TECHNIQUES

FOR KNEC DIPLOMA

MODULE II

QUANTITATIVE TECHNIQUES

Course Outline

QUANTITATIVE TECHNIQUES

GENERAL OBJECTIVES

At the end of this course unit, the trainee should be able to;-

- broaden his/her knowledge in mathematical application;
- understand and appreciate the role of quantitative methods in decision making;
- collect and organize statistical data for management;
- analyze quantitative data for management decision making;
- Apply quantitative methods in solving business problems.

Introduction

Def; quantitative techniques are those techniques which provides the decision maker with a systematic and powerful means of analysis and help, based on quantitative data in exploring policies for achieving pre-determined goals Involves the use of numbers, symbols and other mathematical expressions.
They are essentially helpful in supplementing to judgment and intuition. These techniques evaluate planning factors of alternatives as and when they arise rather than prescribe courses of action. They are particularly relevant to problems of complex business enterprises.
Classification of Q.T

a) Statistical Techniques

Are those techniques which are used in conducting the statistical inquiry concerning a certain phenomenon? They include statistical methods beginning from the collection of data till the task of interpretation of the data collected.

b) Programming Techniques

Are the model building techniques used by decision maker?

Statistical techniques	Programming techniques
Methods of collecting data	Linear programming
Classification and tabulation of collected data	Decision theory
Probability theory and sampling	Theory of games
Correlation and regression analysis	Simulation
Index numbers	a) monte carlo technique
Time series analysis	b) system simulation
Interpretation and extrapolation	queuing (waiting line) theory
	inventory planning

Survey techniques and methodology
Ratio analysis
Statistical quality control
Analysis of variance
Statistical inference and interpretation
Theory of attributes.

network analysis/ PERT
integrated production model
others;
non-linear programming
the theory of replacement
quadratic programming
Parametric programming etc.

QT and Business management

Production Management

- Selecting building site fro a plant, scheduling and controlling its development and designing its layout
- Locating within the plant and controlling movement required materials and finished goods inventories.
- Scheduling and sequencing production by adequate preventive maintenance with optimum number of operatives by proper allocation of machines
- Calculating optimum product mix.

Personnel Management

- Optimum manpower planning
- No of persons to be maintained on permanent or full time roll
- The no. of persons to be kept in work pool intended fro meeting the absenteeism.
- Optimum manner of sequencing and routing of personnel to a variety of jobs
- Studying personnel recruiting procedures, accidents rates and labor turnover

Market Management

- Where distribution and warehousing should be located the size , quantity to be stocked and choice of customers.
- Optimum allocation of sales budget to direct selling of promotional expenses.
- Choice of different media of advertising and bidding strategies.
- Financial management
- Finding long range capital requirement as well as how to generate theses requirements
- Determining optimum replacement policies
- Working out a profit plan for the firm
- Developing capital investment plans
- Estimating credit and investment risks.

Limitation of Q.T s

1. the inherent limitation concerning mathematical expressions
2. high costs involved in the use of QTs
3. They do not take into consideration the intangible factors i.e. non-measurable human factors.
4. Quantitative techniques are just the tools of analysis and not the complete decision making process.

Role of QT in business and industry

1. they provide a tool for scientific analysis

these techniques provides executives with a more precise description of the cause and effect relationship and risks underlying the business operations in measurable terms and this eliminates the conventional
intuitive and subjective basis on which management used to formulate their decisions.

2. they provide solution for various business problems

Are used in the field of production, procurement, marketing, finance and other allied fields. Problems like how best can managers and executive4s allocate available resources to various products so that in a given time the profits are maximized or costs are minimized. Is it possible for an enterprise to arrange the time and quantity of orders of its stocks such that the overall profit with given resources is maximized?

3. they enable proper development of resources

E.g. programmed evaluation and review techniques (PERT) enables us to determine earliest and the latest time fro each of he events and activities and thereby helps in the identification of the critical path
All these helps in deployment of resources from one activity to another to enable the project completion on time.

4. They help in minimizing waiting and servicing time.

The queuing theory helps management in minimizing the total waiting of servicing costs. It also analyses the feasibility of adding facilities and thereby helping to take correct and profitable decision.

5. They enable management to decide when to buy and how much to buy.

The main objective of inventory planning is to achieve balance between the costs of holding stock and benefits of holding stock. Helps in determining when to buy and how much to buy.

6. They assist in choosing an optimum strategy.

In a competitive situation game theory helps to determine optimum strategy which maximizes profits or minimizes loses but adopting optimum strategy.

7. they render great help in optimum resource allocation
8. they facilitate the process of decision making
9. Through various QTs management can know the reactions of the integrated business systems.

CHAPTER ONE

FUNDAMENTALS OF MATHEMATICS AND STATISTICS

Specific Objectives

At the end of this topic, the trainee should be able to:

- Form and solve algebraic equations.
- Apply the various techniques of counting to solving management decision problems;
- Applying set theory to business decision problems;
- Derive and apply the binomial theorem to business problems;
- Evaluate mathematical series.

ALGEBRAIC EQUATIONS

Algebra

Algebra is a branch of mathematics in which, instead of using numbers, we use letters to represent numbers.

We all know that 2+3=5.

Suppose, though, that we substitute letters for the first two numbers, so that:

2 = a

3 = b

We can then write:

a + b = 5

All that has happened is that we have replaced the numbers with letters. However, a number is a specific quantity – e.g., 5 is more than 4, but less than 6 – whereas a letter can be used to represent any number. Thus in the above expression, 'a' could be 4 and 'b' could be 1. We only know that they are 2 and 3 respectively because we defined them as such before.

The main consequence of this is that algebra uses general expression and gives general results, whereas arithmetic (using numbers) uses definite numbers and gives definite results. Arithmetic is specific whereas algebra is general.

Equations

An equation is an expression with an equal sign (=)

Equations are classified into two main groups' linear equations and non linear equations. Examples of linear equations are

x + 13 = 15

7x + 6 = 0

Non linear equations in the variable x are equations in which x appears in the second or higher degrees. They include quadratic and cubic equations amongst others. For example

$5x^2 + 3x + 7 = 0$ (quadratic equation)

$2x^3 + 4x^2 + 3x + 8 = 0$ (cubic equation)

The solution of equations or the values of the variables for which the equations hold is called the roots of the equation or the solution set.

Solution of linear equations.

Supposing M, N, and P are expressions that may or may not involve variables, then the following constitute some rules which will be useful in the solution of linear equations

Rule 1: Additional rule

If M = N then M + P = N + P

Rule 2: Subtraction rule

If M = N, Then M – P = N – P

Rule 3: multiplication rule

If M = N and P ≠ O then M x P = N x P

Rule 4: Division rule

If P x M = N and P ≠ O
And N/P = Q Q being a raterial number then
M = N/P

Example

i. Solve 3x + 4 = - 8

ii. Solve $\frac{y}{3} = -4$

Solutions

i. $3x + 4 = -8$

$3x + 4 - 4 = -8 - 4$ (by subtraction rule)

$3x = -12$ (simplifying)

$\frac{3x}{3} = -\frac{12}{3}$ (by division rule)

$x = -4$ (simplifying)

ii. $3 \times \frac{y}{3} = -4 \times 3$

$y = -12$ (simplifying)

Solutions of inequalities

The solutions sets of inequalities frequently contain many elements. In a number of cases they contain infinite elements.

Example

Solve and graph the following inequalities

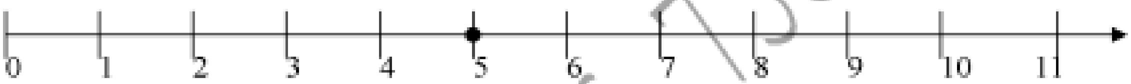
$x - 2 > 2;\ x \subset w$ (where x is a subset of w)

Solution

$x - 2 > 2$ so $x - 2 + 2 > 2 + 2$

Thus, $x>4$

The solution set is infinite, being all the elements in w greater than 4



Example

Solve and graph

$3x - 7 < -13;$

Solution

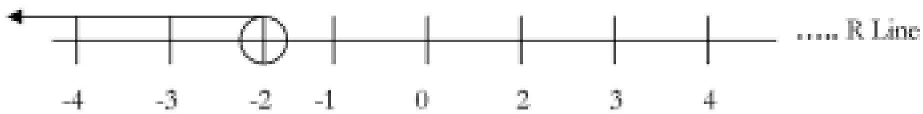
$3x - 7 < -13$

$\Rightarrow 3x - 7 + 7 < -13 + 7$

$\Rightarrow 3x < -6$

$\frac{3x}{3} < \frac{-6}{3}$

$x < -2$



Rules for solving linear inequalities

Suppose M, M_1, N, N_1 and P are expressions that may or may not involve variables, then the corresponding rules for solving inequalities will be:

Rule 1: Addition rule

If $M > N$ and $M_1 > N_1$

Then $M + P > N + P$ and

$M_1 + P > N_1 + P$

Rule 2: Subtraction Rule

If $M < N$ and $M_1 \geq N_1$

Then $M - P < N - P$ and

$M_1 - P \geq N_1 - P$

Rule 3: Multiplication rule

If $M \geq N$ and $M_1 > N_1$ and $P \neq 0$

Then $MP \geq NP$; $M_1P > N_1P$

$M(-P) \leq N(-P)$ and $M_1(-P) < N_1(-P)$

Rule 4: Division

If $M > N$ and $M_1 < N_1$ and $P \neq 0$

Then $M/P > N/P$: $M_1/P < N_1/P$

$M/(-P) < N/(-P)$: and $M_1/(-P) > N_1/(-P)$

Rule 5: Inversion Rule

If $M/P \leq N/Q$ where $P, Q \neq 0$

$M_1/P > N_1/Q$

Then $P/M \geq Q/N$ and $P/M_1 < Q/N_1$

Note: The rules for solving equations are the same as those for solving equations with one exception; when both sides of an equation is multiplied or divided by a negative number, the inequality symbol must be reversed (see rule 3 & Rule 4 above).

Example

Solve and graph the following:

i. $7 - 2x > -11$;

ii. $-5x + 4 \leq 2x - 10$;

iii. $-3 \leq 2x + 1 < 7$;

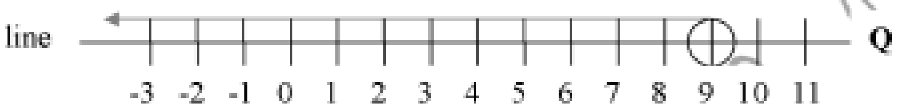
Solutions

i. 7 - 2x > -11

-2x > -18 (subtraction rule)

$\frac{-2x}{-2} < \frac{-18}{-2}$ (bydivision rule)

x < 9

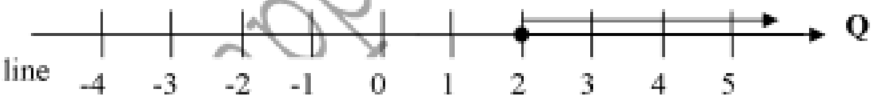


ii. -5x + 4 ≤ 2x - 10

-7x + 4 ≤ -10 (by subtraction rule)

-7x ≤ -14 (by subtraction rule)

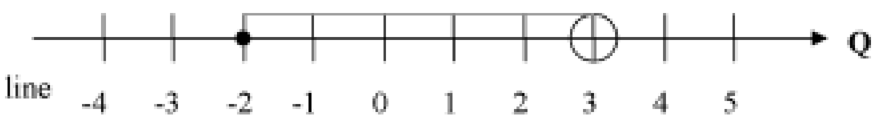
x ≥ 2 (by division rule)



iii. -3 ≤ 2x + 1 < 7

-4 ≤ 2x < 6 (by substraction rule)

-2 ≤ x < 3 (by division rule)



Linear inequalities in two variables: relations

An expression of the form

$$Y \geq 2x - 1$$

Is technically called a relation. It corresponds to a function, but different from it in that, corresponding to each value of the independent variable x, there is more than one value of the dependent variable y
Relations can be successfully presented graphically and are of major importance in linear programming.

Solutions of linear simultaneous equations.

Two or more equations will form a system of linear simultaneous equations if such equations be linear in the same two or more variables.
For instance, the following systems of the two equations is simultaneous in the two variables x and y.

$$2x + 6y = 23$$

$$4x + 7y = 10$$

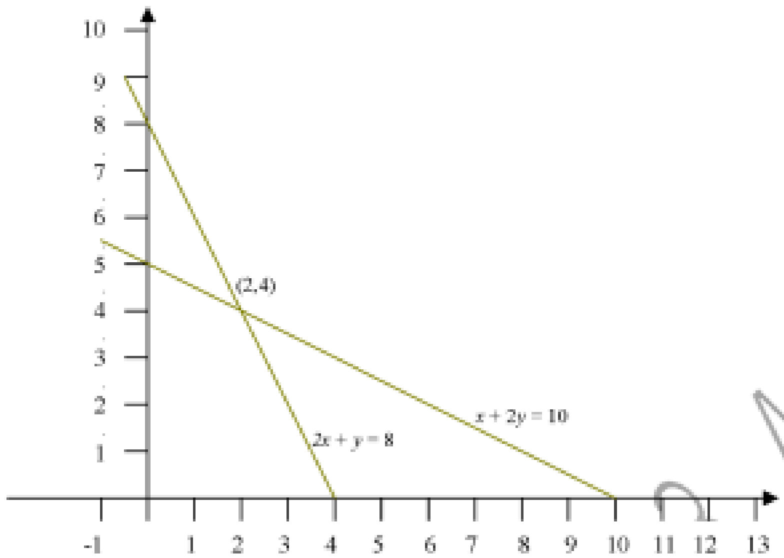
The solution of a system of linear simultaneous equations is a set of values of the variables which simultaneously satisfy all the equations of the system.

Solution techniques

a) The graphical technique

The graphical technique of solving a system of linear equations consists of drawing the graphs of the equations of the system on the same rectangular coordinate system. The coordinates of the point of intersection of the equations of the system would then be the solution.

Example



The above figure illustrates:

Solution by graphical method of two equations

$2x + y = 8$

$x + 2y = 10$

The system has a unique solution (2, 4) represented by the point of intersection of the two equations.

b) The elimination technique

This method requires that each variable be eliminated in turn by making the absolute value of its coefficients equal in the equations of the system and then adding or subtracting the equations. Making the absolute values of the coefficients equal necessitates the multiplication of each equation by an appropriate numerical factor. Consider the system of two equations (i) and (ii) below

$2x – 3y = 8$ …….. ....(i).

$3x + 4y = -5$ …….. ....(ii).

Step 1

Multiply (i) by 3

$6x – 9y = 24$ …… ....(iii).

Multiply (ii) By 2

6x + 8y = - 10 …… ....(iv).

Subtract (iii) from (iv).

17y = -34 ……. ....(v).

∴ y = -2

Step 2

Multiply (i) by 4

8x – 12y = 32 ……. ....(vi)

Multiply (ii) by 3

9x + 12y = -15 ….. ....(vii)

Add (vi) to (vii)

17x = 17 ……..(viii)

∴ x = 1

Thus $x = 1, y = -2$ i.e. {1,-2}

c) <u>The substitution technique</u>

To illustrate this technique, consider the system of two equations (i) and (ii) reproduced below

..... 2x – 3y = 8 ……. (i).

..... 3x + 4y = -5 …… (ii).

The solution of this system can be obtained by

a) Solving one of the equations for one variable in terms of the other variable;
b) Substituting this value into the other equation(s) thereby obtaining an equation with one unknown only
c) Solving this equation for its single variable finally
d) Substituting this value into any one of the two original equations so as to obtain the value of the second variable

Step 1

Solve equation (i) for variable x in terms of y

2x – 3y = 8

x= 4 + 3/2 y (iii)

Step 2

Substitute this value of x into equation (ii). And obtain an equation in y only

$3x + 4y = -5$

$3(4 + 3/2\ y) + 4y = -5$

$8\frac{1}{2}y = -17$ ……. (iv)

Step 3

Solve the equation (iv). For y

$8\frac{1}{2}y = -17$

$y = -2$

Step 4

Substitute this value of y into equation (i) or (iii) and obtain the value of x

$2x - 3y = 8$

$2x - 3(-2) = 8$

$x = 1$

Example

Solve the following by substitution method

$2x + y = 8$

$3x - 2y = -2$

Solution

Solve the first equation for y

$y = 8 - 2x$

Substitute this value of y into the second equation and solve for x

$3x - 2y = -2$

$3x - 2(8-2x) = -2$

$x = 2$

Substitute this value of x into either the first or the second original equation and solve for y

$2x + y = 8$

$(2)(2) + y = 8$

$y = 4$

TECHNIQUES OF COUNTING

Permutations
This is an order arrangement of items in which the order must be strictly observed

Example

Let x, y and z be any three items. Arrange these in all possible permutations

1st	2nd	3rd	
X	Y	Z	
X	Z	Y	
Y	X	Z	Six different permutations
Y	Z	X	
Z	Y	X	
Z	X	Y	

NB: The above 6 permutations are the maximum one can ever obtain in a situation where there are only 3 items but if the number of items exceeds 3 then determining the no. of permutations by outlining as done above may be cumbersome. Therefore we use a special formula to determine such permutations. The formula is given below

The number of permutations of 'r' items taken from a sample of 'n' items may be provided as ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ *where;* ! = factorial

e.g.

i. $${}^{3}P_{3} = \frac{3!}{(3-3)!} = \frac{3 \times 2 \times 1}{0!} = \frac{6}{1} = 6$$

note; 0! = 1

ii. $${}^{5}P_{3} = \frac{5!}{(5-3)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1 \times 2} = 60$$

iii. $${}^{7}P_{5} = \frac{7!}{(7-5)!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{5040}{2} = 2520$$

Example